

Reconstructing the dark energy equation of state with varying alpha

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(Dated: February 5, 2008)

The possibility of reconstructing the dark energy equation of state from variations in the fine structure constant is investigated for a class of models where the quintessence field is non-minimally coupled to the electromagnetic field. For given classes of couplings and quintessence interaction potentials, it is typically found that variations in alpha would need to be measured to within an accuracy of at least 5×10^{-7} to obtain a reconstructed equation of state with less than a twenty per cent deviation from the true equation of state between redshifts 0 and 3. In this case, it is argued that the sign of the first derivative of the equation of state can be uncovered from the reconstruction, thus providing unique information on how the universe developed into its present dark energy dominated phase independent of high redshift surveys. Such information would complement future observations anticipated from the Supernova Acceleration Probe.

PACS numbers: 98.80.Cq

I. INTRODUCTION

Some recent observations of a number of quasar absorption lines indicate that the fine structure constant, $\alpha \equiv e^2/\hbar c$, was smaller than its present value by $\Delta\alpha/\alpha = -10^{-5}$ at redshifts in the range $z \sim 1-3$ [1, 2]. (See, however, Ref. [3, 4] for an independent analysis that does not support such a large variation in α .) Since this redshift range coincides with the epoch when the universe underwent a transition from matter domination to dark energy domination [5, 6], it is of interest to consider the possibility that this change in the effective fine structure constant arises as a direct result of a non-trivial gauge coupling between the dark energy and the electromagnetic field strength [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19] (see also [20, 21, 22, 23, 24, 25, 26, 27, 28]).

In this paper we consider classes of models where the dark energy in the universe is identified as a slowly varying, self-interacting, neutral scalar “quintessence” field [29, 30, 31, 32, 33] (see also the reviews [34, 35]) that is minimally coupled to Einstein gravity but non-minimally coupled to the electromagnetic field. The action is given by

$$S = - \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} R + \int d^4x \sqrt{-g} (\mathcal{L}_\phi + \mathcal{L}_M + \mathcal{L}_{\phi F}), \quad (1)$$

where R is the Ricci curvature scalar of the metric $g_{\mu\nu}$, $g \equiv \det g_{\mu\nu}$, $\kappa^2 \equiv 8\pi m_P^{-2}$ and m_P is the Planck mass. The Lagrangian density for the quintessence field is

$$\mathcal{L}_\phi = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - V(\phi), \quad (2)$$

where $V(\phi)$ is the self-interaction potential of the scalar field, ϕ . The interaction term between the scalar field and the electromagnetic field is determined by

$$\mathcal{L}_{\phi F} = -\frac{1}{4} B_F(\phi) F_{\mu\nu} F^{\mu\nu}, \quad (3)$$

where $F_{\mu\nu}$ is the electromagnetic field strength and $B_F(\phi)$ represents the gauge kinetic function that parametrizes the coupling between the scalar and vector degrees of freedom. \mathcal{L}_M represents the Lagrangian density for the ordinary matter fields and we assume this sector to be dominated by a barotropic pressureless fluid.

The action (1) is characterized in terms of two undetermined functions – the gauge kinetic function and the self-interaction potential. In general, these would be determined by the nature of the underlying particle physics theory. For example, a generic exponential coupling of the type given in Eq. (3) was introduced by Bekenstein [36]. Exponential couplings between form-fields and scalar fields also arise generically in compactifications of string/M-theory to four dimensions, where the scalar field parametrizes the volume of the extra dimensions. (See, e.g., Ref. [37] for a recent review on the theoretical motivation of varying fundamental constants.)

The gauge kinetic function specifies the value of the effective fine structure constant such that $\alpha = \alpha_0/B_F(\phi)$, where a subscript ‘0’ denotes the present-day value. The potential of the field is related to the dark energy equation of state, $w_\phi \equiv p_\phi/\rho_\phi$, where $p_\phi = \dot{\phi}^2/2 - V(\phi)$ represents the pressure of the field, $\rho_\phi = \dot{\phi}^2/2 + V(\phi)$ is the energy density and a dot denotes differentiation with respect to cosmic time.

A number of different approaches may be adopted when fitting models of the form (1) to the data. In principle, the functions $\{V(\phi), B_F(\phi)\}$ would be determined within the context of a unified theory of the fundamental interactions, such as string/M-theory. In this case, a direct approach would be to determine the region of parameter space consistent with observations once these two functions have been specified. This approach was effectively followed recently by Parkinson, Bassett and Barrow [15], who calculated the best fit parameters of a model for which the exact form of the gauge kinetic function and the dark energy equation of state were assumed

a priori.

We adopt an alternative approach in the present work by considering whether the quintessence potential or the gauge kinetic function can be *reconstructed* directly from observational data involving variations in the fine structure constant and the dark energy equation of state. For a given $w(z)$ and $\Delta\alpha/\alpha \equiv (\alpha - \alpha_0)/\alpha_0$, there always exists a $B_F(\phi)$ that would fit the data. Thus, a sufficiently accurate empirical determination of the equation of state, together with the evolution of α , would allow the gauge kinetic function to be reconstructed.

On the other hand, if the gauge kinetic function alone is specified *a priori* (either through theoretical or phenomenological considerations), $w(z)$ and $\Delta\alpha/\alpha$ may no longer be viewed as independent variables, since they share a common origin through the rolling of the quintessence field. In effect, a consistency relation exists between these two quantities and an observational determination of one would constrain the other. This implies that the study of the absorption lines in quasar spectra can in principle yield additional information on variations in the dark energy equation of state (and the corresponding quintessence potential). This is important, given that a determination of the redshift dependence of the dark energy equation of state directly from the luminosity distance relations is difficult – the latter is determined by a double integral over the former and this can severely restrict the available information on the equation of state that can be extracted from observations [38, 39].

In this paper, we consider a linear dependence of the gauge kinetic function on the scalar field:

$$B_F(\phi) = 1 - \zeta\kappa(\phi - \phi_0) \quad (4)$$

where ζ is a constant. This dependence may be viewed as arising from a Taylor expansion of a generic gauge kinetic function and is expected to be valid for a wide class of models when $\kappa(\phi - \phi_0) < 1$ is satisfied over the range of redshifts relevant to observations, $z \approx 0 - 4$. It then follows that the effective fine structure constant depends on the value of the quintessence field such that [9, 14, 40]

$$\frac{\Delta\alpha}{\alpha} \equiv \frac{\alpha - \alpha_0}{\alpha_0} = \zeta\kappa(\phi - \phi_0). \quad (5)$$

Assuming that the mass of the scalar field effectively vanishes, tests of the equivalence principle imply that the parameter, ζ , is bounded by $|\zeta| < 10^{-3}$ [9]. Bounds on variations in the fine structure constant arise from the Oklo natural nuclear reactor ($|\Delta\alpha/\alpha| < 10^{-7}$ at redshift $z = 0.14$ [41, 42] [56]), and the meteorite constraint ($|\Delta\alpha/\alpha| < 10^{-6}$ at redshift $z = 0.45$ [43]).

For consistency, neither the constraint arising from the Oklo natural nuclear reactor nor the meteorite constraint were considered in this work, although it is generally expected that significant variations should be observed up to a redshift of order unity in quintessence models. However, the model in Fig. 3, naturally satisfies the former bound at redshift $z = 0.14$. As discussed in Ref. [40],

these bounds can be evaded through the existence of a form factor in the coupling ζ with respect to the photon momentum. Such a form factor can result in changes in α at the level of atomic physics without leading to observable effects on nuclear phenomena. Moreover, it has been shown for a particular model in Ref. [25], that if dark energy collapses along with dark matter this would naturally lead to a significant difference between the value of the fine structure constant in our galaxy and the one in the background. For these reasons, we decide to explore also the models that do not satisfy the Oklo and meteorites bounds at low redshifts.

II. RECONSTRUCTING THE EQUATION OF STATE: IN PRINCIPLE

To proceed we consider a spatially homogeneous quintessence field propagating in the spatially flat Friedmann-Robertson-Walker (FRW) universe. We assume that the contribution of the electromagnetic degrees of freedom to the total energy density of the universe is negligible and consequently that the cosmic dynamics is determined by the scalar field and a background pressureless fluid (corresponding to dark and visible matter). It then follows that the cosmic dynamics is determined by the Einstein equation

$$HH' = -\frac{\kappa^2}{2}(\rho_M + H^2\phi'^2), \quad (6)$$

and scalar field equation

$$\rho'_\phi = -3H^2\phi'^2, \quad (7)$$

subject to the Friedmann constraint

$$H^2 = \frac{\kappa^2}{3}(\rho_M + \rho_\phi), \quad (8)$$

where a prime denotes differentiation with respect to $N = \ln a$, $H \equiv \dot{a}/a$ is the Hubble expansion parameter, the matter density is given by $\rho_M = \rho_0\Omega_{M0}/a^3$ and ρ_0 denotes the present value of the critical energy density. The term $(dB_F(\phi)/d\phi F_{\mu\nu}F^{\mu\nu})$ containing the derivative of the gauge kinetic function was neglected in Eq. (7) as its statistical average over the present Hubble radius is zero for photons [9].

Only two of the equations (6)–(8) are independent and the cosmic dynamics is fully determined once the functional form of the quintessence potential, $V(\phi)$, has been specified. The nature of the potential determines how the field evolves in time and the corresponding variations in the fine structure constant are then determined by Eq. (5). Moreover, the equation of state is defined in terms of the field's kinetic and potential energies:

$$w \equiv \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)} \quad (9)$$

and its dependence on redshift also follows given the form of the potential.

The principle idea of the inversion procedure is that variations in the fine structure constant may be employed within the context of this class of models to deduce changes in the kinetic energy of the scalar field at higher redshifts. Eqs. (6)–(7) may then be employed to determine the corresponding changes in the field’s energy density, or equivalently, its potential, and hence the equation of state from Eq. (9). This procedure is analogous to that employed in analysing the classical dynamics of a particle, x , moving in a one-dimensional potential well. The form of the well determines the particle’s motion, and this can be represented as a trajectory in the phase space parametrized by $\{x, \dot{x}\}$. Equivalently, the corresponding potential can be reconstructed once the appropriate trajectory has been specified.

We now develop the inversion procedure. Substituting Eq. (8) into Eq. (7) yields a differential equation for the evolution of the energy density of the scalar field:

$$\sigma' = -(\kappa\phi')^2(\sigma + a^{-3}), \quad (10)$$

where we have defined $\sigma = \rho_\phi/\rho_0\Omega_{M0}$. The general solution to Eq. (10) can be expressed in terms of quadratures with respect to the kinetic energy of the quintessence field:

$$\begin{aligned} \sigma(N) = & e^{-\int_0^N dN(\kappa\phi')^2} \times \\ & \times \left[\sigma_0 - \int_0^N dN(\kappa\phi')^2 e^{-3N + \int_0^N dN(\kappa\phi')^2} \right], \end{aligned} \quad (11)$$

where the integration constant σ_0 is defined such that $\sigma(N=0) \equiv \sigma_0 = \Omega_{\phi 0}/\Omega_{M0}$.

The dark energy equation of state is given in general by $w = -1 - (\ln \sigma)'/3$ and substitution of Eq. (10) implies that

$$w(N) = -1 + \frac{(\kappa\phi')^2}{3} \left(1 + \frac{1}{\sigma a^3} \right). \quad (12)$$

Hence, the equation of state can be reconstructed once the redshift dependence of the first derivative of the field has been determined. It is important to emphasize that only the first derivative needs to be measured. If the gauge kinetic function is known, this dependence can be inferred directly from variations in the fine structure constant.

It follows from the definition $\dot{\phi}^2 = \rho_\phi + p_\phi$ that the equation of state (12) can also be expressed in the form

$$w = -1 + \frac{(\kappa\phi')^2}{3\Omega_\phi}, \quad (13)$$

where $\Omega_\phi \equiv \kappa^2 \rho_\phi/(3H^2)$. Differentiating Eq. (13) with respect to scale and substituting Eqs. (6), (7) and (8) then implies that the first derivative (running) of the

equation of state can be expressed directly in terms of first two derivatives of the scalar field:

$$w' = 2(1+w)\frac{\phi''}{\phi'} + 3w \left(1 + w - \frac{(\kappa\phi')^2}{3} \right), \quad (14)$$

where $w' = -(dw/dz)/a$. An infinite hierarchy of expressions relating the n -th derivative of the equation of state to the $(n+1)$ -th derivative of the field could be derived. Each represents a consistency relation between the equation of state and variations in the fine structure constant once the gauge kinetic function has been specified.

We now illustrate the above reconstruction procedure with a specific example where $\kappa(\phi - \phi_0) = \lambda N$ for some constant λ . In this case, the integrals in Eq. (11) can be evaluated analytically:

$$\sigma = \left(\frac{\Omega_{\phi 0}}{\Omega_{M0}} + \frac{\lambda^2}{\lambda^2 - 3} \right) e^{-\lambda^2 N} - \left(\frac{\lambda^2}{\lambda^2 - 3} \right) e^{-3N}, \quad (15)$$

where $\lambda^2 = 3\Omega_{\phi 0}(1+w_0)$. The equation of state is then deduced by substituting Eq. (15) into Eq. (12):

$$w(N) = (\lambda^2 - 3) \left[3 - \frac{\lambda^2}{w_0} \frac{\Omega_{M0}}{\Omega_{\phi 0}} \exp((\lambda^2 - 3)N) \right]^{-1}. \quad (16)$$

Finally, the quintessence potential can be reconstructed by noting that $V(N) = \Omega_{M0}\rho_0\sigma - \phi'^2 H^2/2$ and employing the Friedmann equation (8). We find that

$$V = A e^{-\frac{3}{\lambda}\kappa\phi} - B e^{-\lambda\kappa\phi}, \quad (17)$$

where the mass scales A and B are positive-definite and given by

$$A = \frac{1}{2} \frac{\lambda^2}{3 - \lambda^2} \rho_0 \Omega_{M0} e^{\frac{3}{\lambda}\kappa\phi_0}, \quad (18)$$

$$B = \frac{1}{2} \frac{6 - \lambda^2}{3 - \lambda^2} \rho_0 \Omega_{\phi 0} w_0 e^{\lambda\kappa\phi_0}, \quad (19)$$

respectively. In the above example, it was assumed implicitly that the gauge kinetic function was such that the cosmological variation of $\Delta\alpha/\alpha$ corresponded to a variation in the scalar field of the form $\phi \propto N$. The form of $B_F(\phi)$ was not specified.

III. RECONSTRUCTING THE EQUATION OF STATE: IN PRACTICE

In this section we consider the reconstruction of three dark energy equations of state by employing the method outlined in the previous section for a gauge kinetic function given by Eq. (4). The scalar field potentials have been investigated previously within the context of viable quintessence models [14, 44, 45, 46]. The reconstructions are shown in Figs. 1 – 3. These examples correspond to three different possible evolutions for $w(z)$, namely, those

cases where it increases, decreases or oscillates with increasing redshift. The second and third examples are particularly important as they can not be reproduced with the parametrization employed in Ref. [15], since in that work the equation of state is always increasing with increasing redshift. For the models studied in the present work, we have verified that $\kappa(\phi - \phi_0) < 1$ over the appropriate range of redshifts, and this is consistent with the interpretation of Eq. (4) as a lowest-order Taylor expansion of a generic gauge kinetic function. Let us now describe the reconstruction process.

A. Generating simulated data

In testing the reconstruction procedure, it is necessary to first generate simulated data sets for the variations in α . This was achieved by specifying the functional form of the quintessence potential and numerically integrating the field equations (6)–(8) to determine the redshift dependence of both the equation of state and the quintessence field. The latter determines the corresponding variations in α from Eq. (5) once the coupling parameter, ζ , has been specified. We then generated the simulated data set, with associated error bars, for $\Delta\alpha/\alpha$ based on the exact numerical solution. Specifically, the data points are equally spaced in the redshift range $z \in [0.2, 4]$ at intervals of 0.2 and are normally distributed with mean $\zeta\kappa(\phi - \phi_0)$. In each example, the value of ζ was chosen so that the variations in $\zeta\kappa(\phi - \phi_0)$ resulted in changes in the fine structure constant of the order $\Delta\alpha/\alpha \approx 10^{-5}$, as observed in the present QSO data [2].

B. Fitting the data

The reconstruction can then proceed by fitting the generated data points to a polynomial function

$$g(N) = \frac{\Delta\alpha}{\alpha} = g_1 N + g_2 N^2 + \dots, \quad (20)$$

where g_i are constants. The result of these fits is shown in Fig. 4. Equations (5) and (20) map the variations in α as measured by QSO observations onto the corresponding variations in the scalar field for a given value of the coupling constant, ζ . The degree of the polynomial (20) employed in the fitting differs for the three cases because the underlying equations of state exhibit different levels of complexity. For the class of models we have studied we found that a polynomial of degree three provides generally a good fit to the generated data. However, for models with an oscillating equation of state, one needs to increase the degree of the polynomial in order to obtain both a successful reconstruction and a reduced χ^2 of order unity.

C. Estimating ζ

The first and second derivatives of the field are related to g such that $\phi' = g'/\zeta\kappa$ and $\phi'' = g''/\zeta\kappa$, respectively. In practice, therefore, the numerical value of the coupling parameter must be estimated empirically since the reconstruction via Eqs. (11) and (12) requires the scale dependence of the quintessence field to be known. Substituting Eq. (20) into Eq. (13) implies that

$$\zeta^2 = \frac{1}{3} \frac{g'^2}{\Omega_\phi(1+w)}, \quad (21)$$

and it follows from Eq. (21) that a numerical estimate for the coupling may be deduced given the present-day values of the quintessence field's energy density and the equation of state w_0 , together with the variation, g'_0 , in the fine structure constant, as determined from QSO observations. For example, given the typical values $\Omega_{\phi_0} \sim 0.7$, $-w_0 \sim 0.6 - 0.99$, and $g'_0 \sim 10^{-7} - 10^{-5}$, we find that $\zeta \sim 10^{-7} - 10^{-4}$, in accordance with the values obtained in Ref. [40], where specific quintessence models were studied. This range of values of ζ is compatible with bounds arising from tests of the equivalence principle which demand $|\zeta| < 10^{-3}$ [9].

However, one source of uncertainty in the reconstruction procedure is the uncertainty in the present-day value of the equation of state, w_0 . The latest measurements constrain this parameter within the range $-1.38 < w_0 < -0.82$ at the 95% confidence level, assuming a constant equation of state [47]. This uncertainty generates an uncertainty in the value of the coupling ζ and, in view of this, we allowed the equation of state to take a range of possible values, \tilde{w}_0 . More specifically, in Figs. 1 and 2, the present value of the equation of state was chosen to be $\tilde{w}_0 = w_0 + (-0.1, 0, 0.1)$ where w_0 represents the correct value as deduced from the numerical integration. In Fig. 3, on the other hand, the present value was chosen to be $\tilde{w}_0 = w_0 + (1 + w_0)(0.9, 0, -0.95)$, respectively, when moving downward in the figure.

It follows from Eq. (21) that, when $w \approx -1$, a small uncertainty in the value of \tilde{w}_0 can lead to a large uncertainty in the value of the coupling constant ζ and consequently to distinct possible evolutions for the corresponding equation of state. A typical case of $w \approx -1$ arises when the scalar field undergoes oscillations about the minimum of its potential. On the other hand, an expression equivalent to Eq. (14) for the form of $B_F(\phi)$ adopted in this paper is given by

$$\zeta^2 = \frac{g'^2}{w'} \frac{\Omega_M}{\Omega_\phi} \left(w + \frac{2}{3} \frac{1}{\Omega_M} \frac{g''}{g'} \right). \quad (22)$$

It follows, therefore, that in such cases a more accurate estimate of the magnitude of ζ can be made if information on the present-day value of the first derivative of the equation of state is also available. We note that the quantity dw/dz is an observable parameter believed to

be within reach of future SnIa observations from the Supernova Acceleration Probe (SNAP) [48].

We must note, however, that if ζ was known on fundamental particle physics grounds, the full reconstruction of the equation of state could be achieved without any need to normalize it to an independent result.

D. Reconstruction results

The results of the reconstructions are illustrated in Figs. 1 – 3, where Eq. (21) has been employed to estimate the value of the coupling ζ for the different choices of \tilde{w}_0 by substituting $w \rightarrow \tilde{w}_0$ and $g' \rightarrow g_1$. In each case, the dashed line in the figures represents the exact numerical solution of the equations of motion when $\Omega_{\phi_0} = 0.7$. The corresponding solid lines illustrate the reconstructed evolution for the different values of \tilde{w}_0 considered. An uncertainty in the present value of the equation of state of $\delta w_0 \approx 0.1$ is expected from the SNAP data. Hence, the lines with $\tilde{w}_0 = w_0 \pm 0.1$ (in Figs. 1 and 2) define the error band on the evolution of w arising from the uncertainty we will have on w_0 . It is worth emphasizing that SNAP will provide data within the range of redshifts between 0 and 1.7, whereas the QSO data can (in principle) provide us with information on the equation of state out to a redshift as high as $z = 4$.

When investigating the sensitivity of the reconstruction procedure on errors in the variations of α , we typically found that in order to obtain a reconstruction, $\tilde{w}(z)$, with no more than a 20% deviation from the true equation of state, $w(z)$, i.e.,

$$\frac{\tilde{w}(z) - w(z)}{w(z)} < 0.2, \quad (23)$$

(for $\tilde{w}_0 = w_0$), one requires an observational determination of $\Delta\alpha/\alpha$ to within an accuracy of *at least* $\sim 5 \times 10^{-7}$ between redshifts 0 and 3. Figure 4 shows how small the error bars on data points should be in order to obtain a reliable reconstruction at this order. This is an order of magnitude smaller than the expected sensitivity of the High Accuracy Radial Velocity Planet Searcher spectrograph $\delta(\Delta\alpha/\alpha) \approx 10^{-6}$. [49].

IV. DISCUSSION

Cosmological observations including high redshift surveys of type Ia supernovae and the anisotropy power spectrum of the Cosmic Microwave Background (CMB) indicate that the present-day value of the dark energy equation of state is bounded by $-1.38 < w_0 < -0.82$ at the 95% confidence level, assuming a constant equation of state [47]. Such bounds would be weakened for a wider class of models where the equation of state is allowed to vary, but at present there are only very weak observational constraints on the “running” of the equation of

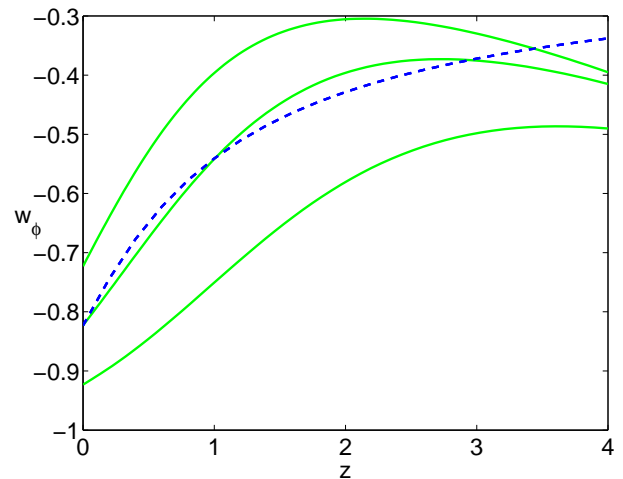


FIG. 1: Dashed line: evolution of the equation of state of the scalar field ϕ with potential $V(\phi) = V_0 \exp((\kappa\phi)^2/2)/\phi^{11}$ determined from numerical integration of the field equations [44]. Solid lines: evolution of the reconstructed equation of state for different possible values of the present-day equation of state (see the text for details). The function $g(N)$ was fitted as a polynomial of degree 3 to a set of points normally distributed with a standard deviation of 1×10^{-7} .

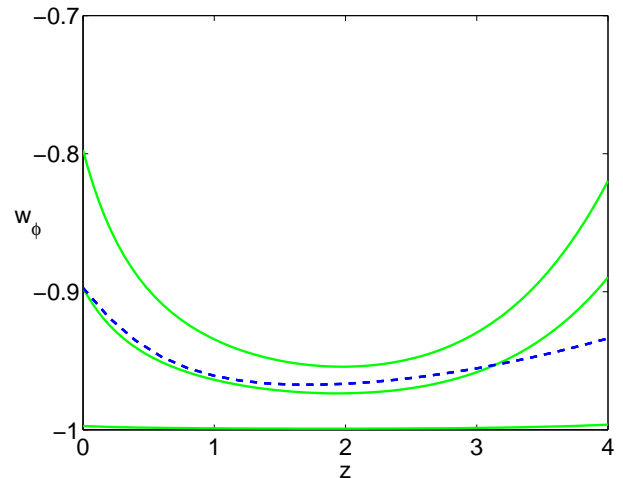


FIG. 2: Dashed line: evolution of the equation of state of the scalar field ϕ with potential $V(\phi) = V_0(\exp(50\kappa\phi) + \exp(0.8\kappa\phi))$ determined from numerical integration of the field equations [45]. Solid lines: evolution of the reconstructed equation of state for different possible values of the present-day equation of state (see the text for details). The function $g(N)$ was fitted as a polynomial of degree 3 to a set of points normally distributed with a standard deviation of 5×10^{-7} .

state, $dw/dz = -aw'$ [50], and it is not yet possible to distinguish such models from a cosmological constant. In this paper, we have investigated the possibility that further information on the redshift dependence of the equation of state can be deduced independently of high redshift surveys through observed variations in the fine structure constant. In principle, the reconstruction of

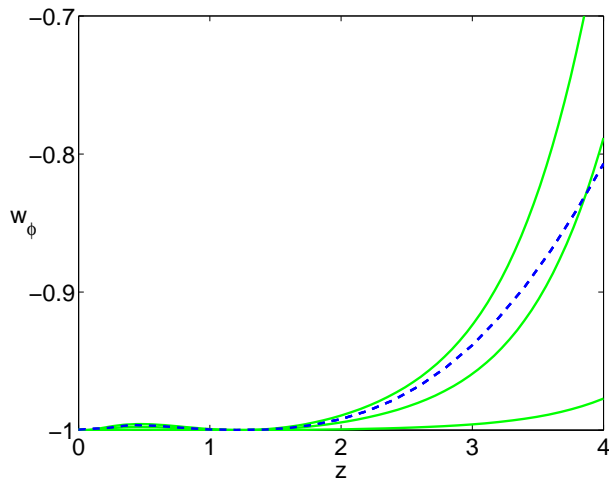


FIG. 3: Dashed line: evolution of the equation of state of the scalar field ϕ with potential $V = \kappa^{-4} e^{-A\kappa\phi} [(\kappa\phi - C)^2 + B]$ determined from numerical integration of the field equations, where $A = 8.5$ and $B = 0.93/A^2$ [14, 46]. Solid lines: evolution of the reconstructed equation of state for different possible values of the present-day equation of state (see the text for details). The function $g(N)$ was fitted as a polynomial of degree 5 to a set of points normally distributed with a standard deviation of 5×10^{-7} .

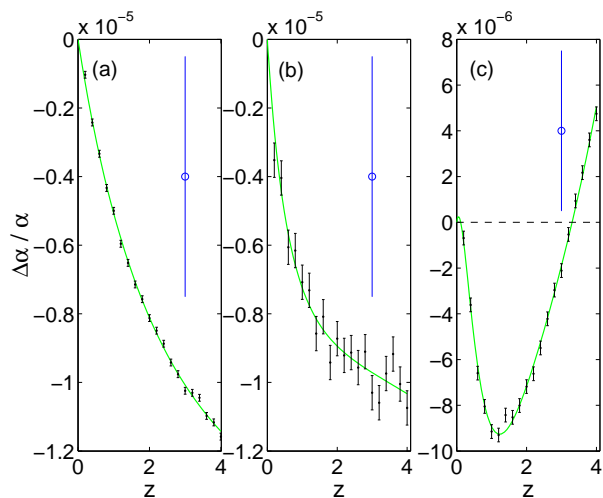


FIG. 4: Evolution of $\Delta\alpha/\alpha$ for the three quintessence models studied in this work. The error bars represent the maximum uncertainty allowed in order to obtain a reliable reconstruction of the equation of state with no more than 20% deviation from that deduced from the numerical integration. (a): $V(\phi) = V_0 \exp((\kappa\phi)^2/2)/\phi^{11}$ [44]; (b): $V(\phi) = V_0(\exp(50\kappa\phi) + \exp(0.8\kappa\phi))$ [45]; (c): $V = \kappa^{-4} e^{-A\kappa\phi} [(\kappa\phi - C)^2 + B]$, where $A = 8.5$ and $B = 0.93/A^2$ [14, 46]. The large error bar in the figures represent a typical current uncertainty on $\Delta\alpha/\alpha$ from QSO observations of $\delta(\Delta\alpha/\alpha) \approx 3.5 \times 10^{-6}$.

the equation of state is possible if the form of the gauge kinetic function that couples the scalar and electromagnetic fields is known. The advantage of a reconstruction of this type is that it yields information on the equation of state at redshifts significantly higher than the limited range accessible to SNAP (corresponding to $z \leq 1.7$).

The primary question addressed in the present paper is how much information one could acquire on $w(z)$ from variations in the fine structure constant alone. In a full reconstruction, one would employ all the data available, from both supernovae surveys and measurements of $\Delta\alpha/\alpha$, and perform a full cross analysis between the different data sets. However, one must also establish what can be learned from each data set independently. Indeed, this is a necessary and crucial step in the program we have outlined, precisely because $w(z)$ and $\Delta\alpha/\alpha$ are not independent as they share a common origin through the quintessence potential. As a result, information on variations in the equation of state determined separately from supernova surveys (see, e.g. [51, 52, 53, 54]) and quasar surveys (as presented above) should be consistent. Establishing an inconsistency would indicate that a reliable reconstruction could not be achieved and, furthermore, would immediately rule out this class of models, namely the form of $B_F(\phi)$, as a mechanism for correlating dark energy and variations in α .

Figs. 1 – 3 indicate that the reconstructions do yield information on whether the running of the equation of state is positive or negative, at least out to a redshift $z \approx 3$. Although the error in the normalization of w_0 implies that the uncertainties in the magnitude of the reconstructed first derivative may be large, the qualitative shape of the equation of state can be deduced, provided variations in α are determined to within an accuracy of 5×10^{-7} or better. We have performed equivalent analyses for the potentials considered in Section III over different regions of parameter space, as well as for other quintessence potentials, and have arrived at similar conclusions.

Any information that can be extracted directly from observations on whether the equation of state increases or decreases with redshift is of importance. For example, if the equation of state increases with redshift (i.e. w moves away from -1), this implies that the field is slowing down as we approach the present day. On the other hand, the kinetic energy of the field is growing as the universe expands if w decreases with increasing redshift. This latter behaviour could correspond, for example, to a creeping quintessence scenario [33], where the field has overshoot the attractor value and has started to move only very recently. Thus, information on the first derivative of the equation of state provides us with unique insight into how the universe underwent the transition from matter domination to dark energy domination.

For the general class of models defined by Eqs. (1) and (5), the qualitative behaviour of the equation of state can be deduced directly from Eq. (21) without the need to solve Eq. (10) if it is observed that g'^2 increases with redshift. Since Eq. (21) is valid over all scales and Ω_ϕ

is a decreasing function of redshift, it necessarily follows that the equation of state must have been larger in the past and this case would therefore rule out the possibility of a creeping quintessence scenario. On the other hand, for the case where g'^2 is a decreasing variable, we must proceed to solve Eq. (10) directly in order to gain further insight.

Finally, we outline a complementary approach that may allow the equation of state and its derivatives to be deduced at a specific redshift. This approach corresponds to a perturbative reconstruction of the equation of state. It follows from Eqs. (14) and (5) that the first derivative of the equation of state at a given redshift can be directly determined if the corresponding values of $\{w, g', g''\}$ are known. Higher derivatives can also be constrained if sufficient information on the corresponding derivatives of the fitting function $g(N)$ is available. Assuming that the necessary constraints on the derivatives could be determined from QSO observations, the one re-

maining free parameter would be the equation of state, or equivalently from Eq. (13), the density of the dark energy. This parameter could in turn be deduced from the Friedmann equation (8) if the Hubble parameter, $H(z)$, were known and this could be found from the luminosity distance, d_L :

$$H^{-1}(z) = \frac{d}{dz} \left(\frac{d_L}{1+z} \right) \quad (24)$$

It would be interesting to explore this possibility further.

Acknowledgments

NJN is supported by the Particle Physics and Astronomy Research Council (PPARC). We thank C. Martins and M. Pospelov for helpful comments and suggestions.

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